Radio Propagation Modeling via Fourier Optics

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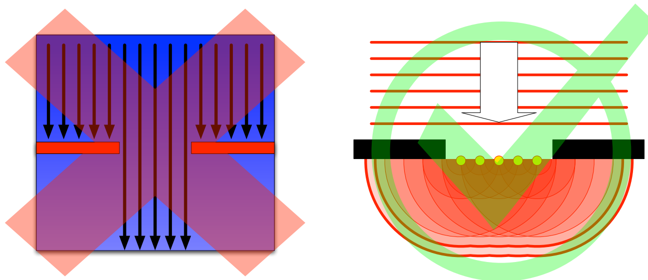

Abstract

Although Hugens published his theory on optical wavefronts in 1690 [Huygens 1690] detailing how light propagates, within the graphics community ray optics have remained the model of choice for how light behaves. While this is acceptable for visible light, where objects that are visible to the naked eye are usually large in comparison to the wavelength of the light illuminating them and therefore diffract very little, it is unacceptable for radio propagation, whose wavelengths are on the order tens of centimeters to many kilometers, and which therefore diffract strongly. In order to improve on the quality of simulation currently available, the author implemented the methods detailed in [Ahrenberg et al. 2008] on a GPU as a first step towards accurate modeling of radio propagation. This paper details the current state of the author's experience in implementing these methods.

CR Categories: I.3.m [Computer Graphics]: Miscellaneous—Fourier Optics;

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1 Introduction

All models are wrong. Some models are useful.
(George E. P. Box)

Graphics has used simple, ray-based optical models for many reasons, including the following:

- **Simplicity** - Rays are intuitively simple to understand, and are therefore easy to implement.
- **Speed** - With simple scenery, intersections are simple to calculate, which makes them fast to calculate.
- **Reasonable accuracy** - Although the model is wrong, it is 'good enough for graphics work'. The model doesn't look *too* wrong, so the errors are forgiven.

Unfortunately, while these reasons may be acceptable in the region of the electro-magnetic spectrum that is visible to human beings, the last point in particular is wrong in the radio region. In this region, diffraction is a serious problem because radio wavelengths are on the order of centimeters to kilometers in length, which happens to be the rough scale of human beings and objects humans regularly interact with. This means that while a flashlight's output can be easily modeled as a simple cone intersecting objects in a scene, radio waves will naturally tend to bend (diffract) around objects in the scene. This is part of the reason why two radios on opposite sides of a mountain can sometimes connect to each other; depending on the mountain's shape and geology, it may be 'sharp' in relation to the radio waves propagating over it, causing them to bend down into the valley below.

This problem becomes much worse when the scenes that are being modeled are enclosed, conductive, reflective, and on the order of the length of the radio waves being transmitted through them. A good example of this problem involves cellphones and GPS, both of which operate on frequencies with wavelengths between 10 cm and 1 meter. It is not uncommon for cellphone reception to be poor within buildings, and for GPS to work poorly in regions with tightly packed buildings ('urban canyons'). This is due to the multi-path problem, which is a particular example of constructive and destructive interference; thanks to the interference, moving a short distance within a building may improve communications significantly, or may destroy it completely. Note that this interference is exacerbated by the narrow bandwidth that typical communications use. For example, ordinary FM radio in the United States has a carrier frequency of 87.5 and 108.0 MHz, but each station is restricted to a bandwidth of 15 kHz (see [FCC 2005] for the FCC regulations). Because the bandwidth is so narrow, this is almost a single frequency; if this were a wide band, then it would be possible that while some frequencies interfered destructively at a particular location, others might interfere constructively, permitting communication along some band.

Diffraction and multi-path interference, when taken together, become almost insurmountable for the ray-optics model; not only is the model wrong, it is no longer useful.

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1.1 Fourier Optics

Since ray-optics are no longer useful, it is necessary to develop a new model which can overcome this limitation. Fortunately, the answer lies in one of the causes of the problems; the narrow bandwidth of the radio transmitters in use. They are essentially single frequency devices. In addition, if the radio antennae is a simple dipole or monopole, then the output will be naturally polarized along the antennae. If we are willing to accept some loss in accuracy, then we can model the interference in the far-field as a coherent, polarized light source, similar to what a laser or a maser might produce.

This observation lead the author to an interesting comparison to how holograms are produced, and from there, computational holography, for which there are numerous resources (see [Ahrenberg et al. 2008; Voelz 2011; Goodman 2004]). Fourier optics, which is used to compute the holograms in computational holography, is able to correctly model diffraction, and when coupled with accurate models of a scene may be able to give good results for radio propagation. For this reason, the author is exploring its application to radio propagation modeling.

1.2 Paper Organization

The rest of this paper is organized as follows:

- §2 Outlines the mathematical model used by the author, as well as some of its limitations
- §3 Provides timing results on an NVIDIA GeForce 9600M GT
- §4 Outlines what work needs to be done
- §5 Conclusion

2 Method

2.1 Mathematical Model

Fourier methods depend on the Fourier transform (Eq. 1) and the Convolution Theorem (Eq. 2):

$$(L \circledast S)(\mathbf{x}) = \int_{\mathbb{R}^{|\mathbf{x}|}} L(\mathbf{y})S(\mathbf{x}-\mathbf{y})d\mathbf{y} \tag{1}$$

$$\mathcal{F}(L \circledast S) = \mathcal{F}(L) \bullet \mathcal{F}(S) \tag{2}$$

In essence, if we have a model of the light L that is illuminating a scene S, then in normal space we would convolve the two together. However, the Convolution Theorem states that any convolution in normal space can be calculated as the multiplication of the Fourier transforms of the functions in Fourier space. The advantage of this is the light model and the scene model are no longer linked; one can calculate and cache the scene model separately from the light model. Even greater speedups can be achieved by manipulating the light and the scene in Fourier space directly; in that case, the calculation becomes one of selecting a cached object template, performing an affine transform to manipulate it into the form we need, and then performing the convolution. Bracewell et. al. showed in [Bracewell et al. 1993] that it is possible to perform an affine transform in 2D Fourier space. Ahrenberg et. al. showed in [Ahrenberg et al. 2008] how this can be applied to compute triangles when performing computational holography. The equations from [Ahrenberg et al. 2008] (with minor changes and corrections) are shown below:

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \end{bmatrix} \text{ A rotation matrix} \tag{3}$$

$$d(u,v) = \sqrt{\frac{1}{\lambda^2} - u^2 - v^2} \tag{4}$$

$$J = \begin{array}{l} (r_{12}r_{23} - r_{13}r_{22})\frac{u}{d(u,v)} + \\ (r_{13}r_{21} - r_{11}r_{23})\frac{v}{d(u,v)} + \\ (r_{11}r_{22} - r_{12}r_{21}) \end{array} \tag{5}$$

$$\hat{u} = r_{11}u + r_{12}v + r_{13}d(u,v) \tag{6}$$

$$\hat{v} = r_{21}u + r_{22}v + r_{23}d(u,v) \tag{7}$$

$$A'(u,v) = A(\hat{u},\hat{v})J(u,v) \tag{8}$$

$$A''(u,v) = A'(u,v)\exp\left(\frac{2\pi i}{\lambda} r\sqrt{1-(u\lambda)^2-(v\lambda)^2}\right) \tag{9}$$

where λ is the wavelength of the spectrum illuminating the scene, $A(u,v)$ is the Fourier transform of some set of 2 dimensional objects within their own plane, and r is the distance separating the plane containing the objects from the plane that is being illuminated by light reflected off of the objects. Note that r is calculated only after the object plane is rotated to be parallel to the illuminated plane.

Ahrenberg et. al. realized in [Ahrenberg et al. 2008] that an affine transform could be applied to a single template object in Fourier space, allowing it to be stretched into any other object of the same form. By applying this to a triangle template, they could generate exact solutions for any triangle without having to recalculate the Fourier transform. This resulted in the following equations (with minor changes) for their template triangle, whose corners on some plane in Fourier space are at $(0,0)$, $(1,0)$, and $(1,1)$:

$$F_\Delta(u,v) = \begin{cases} \frac{1}{2} & u = v = 0 \\ \frac{1-\cos(-2\pi v)}{(2\pi v)^2} - i\frac{2\pi v - \sin(-2\pi v)}{(2\pi v)^2} & u = 0, v \neq 0 \\ \frac{\cos(-2\pi u) - 2\pi u \sin(-2\pi u) - 1}{(2\pi u)^2} + i\frac{2\pi u\cos(-2\pi u) + \sin(-2\pi u)}{(2\pi u)^2} & u \neq 0, v = 0 \\ \frac{1-\cos(-2\pi v)}{(2\pi v)^2} + i\frac{2\pi v - \sin(-2\pi v)}{(2\pi v)^2} & u = -v, v \neq 0 \\ \frac{(u+v)\cos(-2\pi u) - v - u\cos(-2\pi(u+v))}{(2\pi)^2 uv(u+v)} + i\frac{(u+v)\sin(-2\pi u) - u\sin(-2\pi(u+v))}{(2\pi)^2 uv(u+v)} & \text{else} \end{cases} \tag{10}$$

With the template triangle in hand, it is possible to generate the Fourier transform of any triangle by performing an affine transform on the template. To do so, let's define a general triangle as having the points (s_1,t_1), (s_2,t_2), and (s_3,t_3). We can now define the mapping:

$$\begin{aligned} (0,0) &\mapsto (s_1, t_1) \\ (1,0) &\mapsto (s_2, t_2) \\ (1,1) &\mapsto (s_3, t_3) \end{aligned} \tag{11}$$

and the affine transformation:

$$\begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix} \tag{12}$$

That leads to the following equations (see [Bracewell et al. 1993] and [Ahrenberg et al. 2008])

$$a_{11} = t_3 - t_2 \tag{13}$$

$$a_{12} = s_2 - s_3 \tag{14}$$

$$a_{13} = -((t_3 - t_2)s_1 + (s_2 - s_3)t_1) \tag{15}$$

$$a_{21} = t_1 - t_2 \tag{16}$$

$$a_{22} = s_2 - s_1 \tag{17}$$

$$a_{23} = -((t_1 - t_2)s_1 + (s_2 - s_1)t_1) \tag{18}$$

$$J = a_{11}a_{22} - a_{12}a_{21} \tag{19}$$

All of this culminates in [Ahrenberg et al. 2008]:

$$\begin{aligned} \alpha &= a_{22}a_{13} - a_{12}a_{23} \\ \beta &= a_{11}a_{23} - a_{13}a_{21} \\ \eta &= \tfrac{a_{22}u - a_{12}v}{J} \\ \sigma &= \tfrac{-a_{12}u + a_{11}v}{J} \\ F_\Gamma &= \tfrac{1}{|J|} \exp\left\{ \tfrac{2\pi i}{J} [\alpha u + \beta v] \right\} \times F_\Delta(\eta, \sigma) \end{aligned} \tag{20}$$

where α, β, η, and σ have been introduced only because of formatting constraints in this paper.

Using these equations, we can work exclusively in Fourier space until it is time to extract an answer. At that time, we need to perform a discrete inverse Fourier transform, which will reduce our accuracy somewhat, but because the results were exact to this point, our accuracy is still greater than it would have been if we had to do the discrete forward transform first.

2.2 Limitations

The primary limitation of the method outlined in [Ahrenberg et al. 2008] and used here is that it assumes a perfectly uniform illumination model, and a perfectly diffuse surface; this is not a limitation of the Fourier method, but merely a limitation of the math currently used. A limitation of the current method outlined above is that it does not model reflection or polarization; since this is critical for modeling radio propagation, it must be included before this method can be used.

3 Results

Currently, because this method does not model either reflection or polarization (see §2.2), the method cannot be verified with a ground-truth model; only the timing results can be presented. For this reason, although the author believes that this method holds great promise, he is not recommending its general use at this time. The ideas outlined here should only be used as ideas for future work.

That said, a GPU accelerated version of this work took on average 19.215 ms to process 1000 triangles on a NVIDIA GeForce 9600M GT. This is more than sufficient for many applications, including the author's primary area of interest, robotics, where predictions of radio propagation need only occur at approximately 10 Hz to be useful.

4 Future Work

Future work includes modeling all of the following:

- Reflection
- Polarization
- Absorption & Transmission
- Refraction
- Code optimization

And, of course, evaluation against some known ground-truth.

5 Conclusion

Calculating illumination in Fourier space has a great many advantages, not the least of which is that it is more accurate than calculating the results in normal space. However, before it can supplant ray optics as a model of how light behaves, work must be done to improve the models so that the deficiencies outlined in §4 are properly addressed. The author believes all of these problems are surmountable. He is continuing to work on improving the mathematical models, as well as increasing the efficiency with which they are calculated.

References

AHRENBERG, L., BENZIE, P., MAGNOR, M., AND WATSON, J. 2008. Computer generated holograms from three dimensional meshes using an analytic light transport model. *Appl. Opt. 47*, 10, 1567–1574.

BRACEWELL, R. N., CHANG, K. Y., JHA, A. K., AND WANG, Y. H. 1993. Affine theorem for two-dimensional Fourier transform. *Electronics Letters 29*, 3, 304+.

FCC. 2005. *TITLE 47–Telecommunication, CHAPTER I–FEDERAL COMMUNICATIONS COMMISSION, SUBCHAPTER A–GENERAL, PART 2–FREQUENCY ALLOCATIONS AND RADIO TREATY MATTERS; GENERAL RULES AND REGULATIONS*, vol. 70 FR 580. Federal Register, January.

GOODMAN, J. W. 2004. *Introduction to Fourier Optics*, third ed. Roberts & Company Publishers, December.

HUYGENS, C. 1690. *Traitė de la Lumiere*. Pieter van der Aa, Leiden, Netherlands.

VOELZ, D. G. 2011. *Computational Fourier Optics: A MATLAB Tutorial*, vol. TT89 of *SPIE Tutorial Texts*. SPIE Press, January.