The Undefined Domain: Precise Relational Information for Entities that Do Not Exist

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Static Analysis in the Presence of Dynamic Memory

Abstract Interpretation

- *domains* approximate sets of program states
- numeric domains hold numeric valuations for variables (state $s \subseteq \chi(s) \to \mathbb{Z}$ where $\chi(s)$ is the support set of s)
- e.g. *Intervals* domain gives non-relational approximation (state $s : \chi(s) \to \mathcal{I}$)
- *relational* domains infer relations between variables, e.g.

  Polyhedra (linear inequalities)

  TVPI (two variables per inequality)

Challenge: Joining states with different support sets

dynamic memory

how to represent regions that only exist in some traces

uninitialized values

how to avoid precision loss in presence of uninitialized ($\top$) values

Scenario 1: Non-existing fields

```
int * p;
if(rnd()) {
  p = malloc(4);
  *p = 3;
  // [p = 1 · &x, x = 3]
} else {
  p = NULL;
  // [p = 0 · &x]
}
// [p = [0, 1] · &x, x = ?]
```

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   // [p = 1 · &x, x = 3]
} else {
   p = NULL;
   // [p = 0 · &x, x = ⊥]
}
// [p = [0, 1] · &x, x = 3]
```

Attempt 1: add $\bot$ value

- $\bot \equiv \emptyset$
- most precise
- can be done in *Intervals* domain

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- most precise
- can be done in *Intervals* domain

But:

- not possible when domain models *smash product*:
  no "individual" $\bot$ value for x
- relational domains usually model smash products:
  rules out Polyhedra, TVPI, Octagon, ...

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```

// $[p = [0,1] \cdot \&x,\ x = \begin{cases} \top & \text{if } p = 0 \\ 3, & \text{otherwise} \end{cases}]$

Attempt 2: add $\top$ value

- here, $\top \equiv \mathbb{Z}$
- can be represented in relational domains

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Attempt 2: add $\top$ value

- here, $\top \equiv \mathbb{Z}$
- can be represented in relational domains

But:

- no linear inequality can express relation between p and x
- Approximating state in Polyhedra domain gives

$$[p \in [0,1] \cdot \&x,\ x = \top]$$

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Attempt 3: *copy and paste* value

- copy value from other branch
- paste value to missing field

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Success

- can be used in relational domains
- copied value "fits into" joined state
- avoids precision loss
- but: only sound when pasted value is not reachable

Scenario 2: Non-initialized fields

```
int x, y;
  // [x = ⊤, y = ⊤]
  //
if(rnd()) {
  x = 1;
  y = 2;
  // [x = 1, y = 2]
  //
} else {
  x = 0;
  // [x = 0, y = ⊤]
  //
}
// [x ∈ {0,1}, y = { ⊤  if x = 0 ; 2, otherwise }]
//
```

Precision loss

- final state approximated in Polyhedra domain:

$$[x \in \{0,1\},\ y = \top]$$

- and we cannot copy and paste...

Scenario 2: Non-initialized fields

```
int x, y;
  // [x = ⊤, y = ⊤]
  // []
if(rnd()) {
  x = 1;
  y = 2;
  // [x = 1, y = 2]
  //
} else {
  x = 0;
  // [x = 0, y = ⊤]
  //
}
// [x ∈ {0,1}, y = { ⊤ if x = 0 ; 2, otherwise }]
//
```

Precision loss

- final state approximated in Polyhedra domain:

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Solution: Avoid $\top$ as value

- do not initialize variables with $\top$
- copy and paste missing variables when joining
- flags indicate definedness
  $1 \approx$ value given by domain state
  $0 \approx$ value is unrestricted

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  y = 2;
  // [x = 1, y = 2]
  // [x = 1, f_x = 1, y = 2, f_y = 1]
} else {
  x = 0;
  // [x = 0, y = ⊤]
  //
}
// [x ∈ {0,1}, y = { ⊤  if x = 0 ; 2, otherwise }]
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} else {
  x = 0;
  // [x = 0, y = ⊤]
  // [x = 0, f_x = 1, y = 2, f_y = 0]
}
// [x ∈ {0,1}, y = { ⊤  if x = 0 ; 2  otherwise }]
//
```

Precision loss

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  // [x = 1, y = 2]
  // [x = 1, f_x = 1, y = 2, f_y = 1]
} else {
  x = 0;
  // [x = 0, y = ⊤]
  // [x = 0, f_x = 1, y = 2, f_y = 0]
}
// [x ∈ {0,1}, y = { ⊤ if x = 0 ; 2, otherwise }]
// [x ∈ {0,1}, f_x = 1, y = 2, f_y = x]
```

Precision loss

- final state approximated in Polyhedra domain:

$$[x \in \{0,1\},\ y = \top]$$

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Solution: Avoid $\top$ as value

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Putting things together: The *Undefined* Domain

Functor Domain

denoted by $U(D)$ for some child domain D

state $u \rhd s \in U(D)$ where $s \in D$

Support set χ

$x \in \chi(u \rhd s) \Longleftrightarrow x,\ f_x \in \chi(s)$

Semantics

if $f_x = 1$ use value of x

if $f_x = 0$ treat x as undefined

Putting things together: The *Undefined* Domain

Functor Domain

denoted by $U(D)$ for some child domain D

state $u \triangleright s \in U(D)$ where $s \in D$

Support set χ

$x \in \chi(u \triangleright s) \iff x,\ f_x \in \chi(s)$

Semantics

if $f_x = 1$ use value of x

if $f_x = 0$ treat x as undefined

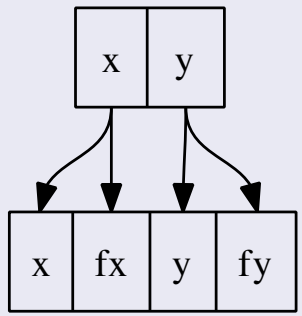
From the last example

represented state $u \triangleright s$

$$u \triangleright s \equiv [x \in \{0,1\},\ y = \begin{cases} \top & \text{if } x = 0 \\ 2, & \text{otherwise} \end{cases}]$$

child state s

$s = [x \in \{0,1\},\ f_x = 1,\ y = 2,\ f_y = x]$



Lattice Operations: Join and Compare

- make *compatible*: adjust support sets before joining and comparing
- make a "good guess" for missing values:
  we *copy and paste* values from other state
- copying values én bloc retains relational information

Comparing States: $u \rhd s \sqsubseteq u \rhd t$

- make states compatible
- then perform comparison $s \sqsubseteq t$ on child domain

Joining States: $u \rhd s \sqcup u \rhd t$

- make states compatible
- then perform join $s \sqcup t$ on child domain

Lattice Operations: Copy and Paste

Making states $u \rhd s$ and $u \rhd t$ compatible

first add variables missing in right hand side $u \rhd t$:

1. compute set $\chi_{\text{common}} := \chi(s) \cap \chi(t)$ of common variables
2. copy values missing in t from s:
   $L := drop_{\chi_{\text{common}}}(u \rhd s)$
3. paste values to t:
   $t := t \times L$
4. set flags to "undefined" for added variables:
   for each non-flag variable $x \in L$: $t := t[\![f_x := 0]\!]$

then same for left hand side

Lattice Operations: Correctness

Co-fibered Domain

- Pre-order $\sqsubseteq$ defined by Grothendieck construction:
  for each set χ of variables:

  projection $drop_\chi$

  $$u \rhd s \sqsubseteq drop_\chi(u \rhd s)$$

  family of introduction operations add_χ

  $$u \rhd s \sqsubseteq add_\chi(u \rhd s)$$

- ensures soundness of making states compatible

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- ensures soundness of making states compatible

Transfer Functions 1: Assignments

assignment operation

for example $x := y + z$

(note that $+$ is a total function)

perform operation on values

on child domain: $x := y + z$

sound, because operation $+$ is total

adjust flags

on child domain: $f_x := f_y \wedge f_z$

result is undefined if an operand is undefined

Transfer Functions 1: Assignments

Caution: non-total functions (e.g. division)

```
if(rnd()) {
   x=1; y=0;
} else {
   x=0;
}
// [x = [0,1], f_x = 1, y = 0, f_y = x]
if(x==0) {

   y=3/y;

}
```

Transfer Functions 1: Assignments

Caution: non-total functions (e.g. division)

```
if(rnd()) {
   x=1; y=0;
} else {
   x=0;
}
// [x = [0,1], f_x = 1, y = 0, f_y = x]
if(x==0) {
   // [x = 0, f_x = 1, y = 0, f_y = 0]
   y=3/y;
   // [⊥]
}
```

Transfer Functions 2: Tests

Tests, for example $x + y < 0$

split child state

defined arguments

$$s_d = s[\![f_x = 1 \wedge f_y = 1]\!]$$

undefined argument

$$s_u = s[\![f_x = 0 \vee f_y = 0]\!]$$

perform test only where values are defined

$$s := s_u \sqcup s_d[\![x + y < 0]\!]$$

Efficient Implementation

Sources of Infficiency

one flag per variable, although

- many flags have constant value
- join operation introduces flags of equal value

How to avoid them

- do not store constant flags in child domain
- store only one flag for groups of equal-valued flags

Achieved improvement

ratio between number of variables and flags becomes better than 10 : 1

Experimental Results

	U	steps	time	memory	vars	add. flags	warnings
heap 1	✓	12	19	19.0	14	1	0
	✗	12	18	17.8	13	–	1
heap 2	✓	24	35	23.2	23	2	0
	✗	24	32	21.7	21	–	1
call stack 1	✓	114	450	42.0	50	2	0
	✗	76	377	41.9	48	–	7
call stack 2	✓	254	641	42.0	74	2	0
	✗	178	416	42.6	72	–	7
call stack 3	✓	153	718	42.4	66	4	0
	✗	76	422	41.9	48	–	7
call stack 4	✓	128	702	42.2	54	2	0
	✗	88	920	42.5	52	–	8
call stack 5	✓	173	1455	47.3	75	4	0
	✗	90	709	42.0	54	–	8

- the *Undefined* domains improves precision
- only a small number of flags is required

Finally...

the Undefined domain

provides a simple solution to a common problem

- enhances expressiveness of existing domains
- combination of two simple techniques:
  - mask "undefined" values by flags
  - copy and paste operation that respects relational information
- clean theoretic presentation as a co-fibered domain

related work

- recency abstraction [Balakrishnan et. al. 2006]
  most recent instance of object is kept apart from summary
- decision tree domain (Astrée [Cousot et. al., 2003])
  tree of domains, selected by flag value

future work

explore operations that retain even more relations than *copy and paste*