CosmoSIS Webinar Part 1

An introduction to cosmological parameter estimation and sampling

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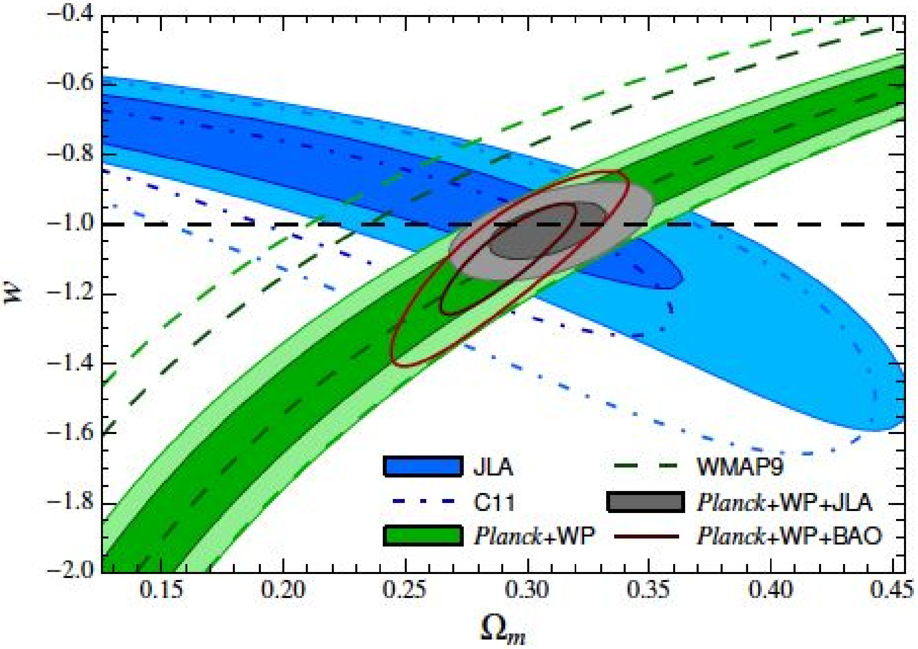
Goal of this webinar



- How do I generate images like this if I have some data ?
- How do I combine datasets to produce different contours ?
- If a sampler is performing poorly, can I use a different one ?

Overview

- **Models and parameter spaces**
- CosmoSIS
- Sampling methods
- Metropolis Hastings theory & example in CosmoSIS
- Making plots from MCMCs

Cosmological parameters

$$\{A_s, n_s, H_0, \Omega_m, \Omega_b, \tau\}$$

Given the precision of our current and future experiments, we rarely can use analytical solutions to predict observables!

Calculations are getting increasingly complicated.

Examples:

CMB: Require Boltzmann Codes, e.g. CAMB

LSS: Need codes to model non-linear corrections to the matter power spectrum, e.g. Halofit, emulators

Model	Parameter	General meaning	Needed for
Flat LCDM	Omega_m	Matter density fraction	Background evolution
	H_0	Expansion rate	Background evolution
	Omega_b	Baryon density fraction	Thermal history
Adding parameters to the model	A_s, sigma_8	Variance of cosmic density structure	Structure formation
	n_s	Structure scale-dependence power law	Structure formation
LCDM	Omega_k	Curvature	Background evolution
nuLCDM	Omega_nu	Neutrino density fraction	Small-scale structure
	massive_nu	Number of massive neutrinos	Small-scale structure
	massless_nu	Number of massless neutrinos	Small-scale structure
wCDM	w	Dark Energy constant equation of state	Background evolution
Extended DE	w_a, w_p, ...	Dark Energy varying equation of state	Background evolution

Nuisance parameters

Most datasets require additional “Nuisance parameters”

These can describe

- Data-specific physical effects (e.g.: supernova light curve standardization)
- Foregrounds (e.g. dust foreground parameters)
- Parameters for instrument systematics (e.g. calibration parameters)

Example: The Planck 2015 likelihood can have 16 different nuisance parameters:

https://wiki.cosmos.esa.int/planckpla2015/index.php/CMB_spectrum_%26_Likelihood_Code

Parameter spaces

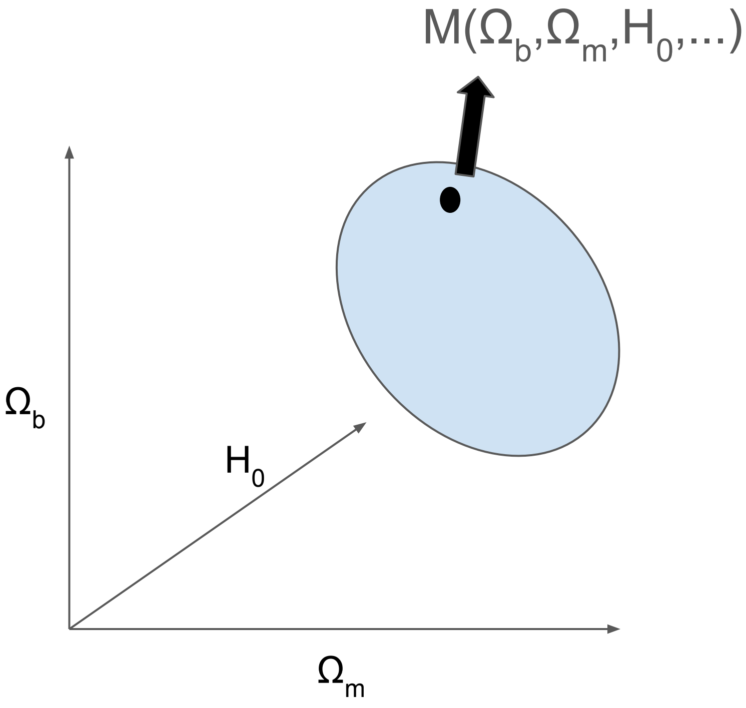
Inference takes place in a multi-dimensional parameter space.

-can be highly non-Gaussian!

Given a model, M: which regions of parameter space are reasonable fits to the data?

Probabilities in this space are functions of many variables: $P(\Omega_b,\Omega_m,H_0,...)$

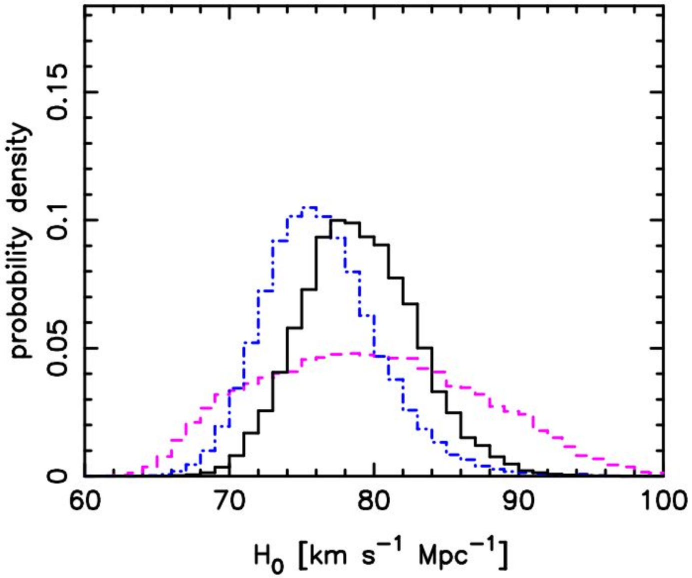
Caution: Intuition becomes bad as the dimension of the parameter space increases.



Does $M(\Omega_b,\Omega_m,H_0,...)$ resemble the data at a given point?

Parameter estimation: what does a sampler output?

- H_0 = 78 km/s/Mpc ✖ Best fit only
- H_0 = (78 ± 8) km/s/Mpc ✖ simple error bar (Max likelihood methods)
-
- H_0 =



Suyu et al (2016)

A true measurement of a parameter is a probability function showing its **Posterior probability distribution function**.

Often summarize distributions using just mean and variance

This is only a complete description in special cases like Gaussian distributions

Bayes Theorem

Likelihood: we must determine this function of the model and data from the physics of the problem

Prior: previous information

$$P(\text{params}|\text{data, model}) = \frac{L(\text{data}|\text{params, model}) \quad P(\text{params}|\text{model})}{P(\text{data}|\text{model})}$$

Posterior distribution for the parameters: the thing we want!

Evidence: normalizing constant (for now; also used to compare entire models)

$$p(\theta|D, M) = \frac{\mathcal{L}(D|\theta, M)\pi(\theta|M)}{p(D|M)}$$

Set of parameters

Constraining cosmological parameters given data

- Step 1: At given point in parameter space: generate a prediction $M(\Omega_b , \Omega_m, H_0,\ldots)$.
  - Example: compute CMB power spectrum
- Step 2: Compute a probability of the data given that prediction
  - Example: Use Planck likelihood
- Step 3: Use Bayes Theorem to get the posterior given some prior
  - Example: use local measurements of H0 as a prior
- Step 4: Sampler proposes next point in the parameter space to evaluate likelihood
  - Example: MCMC step through parameter space using proposal distribution

Given the precision of current experiments, developing software for steps 1 and 2 can be quite challenging.

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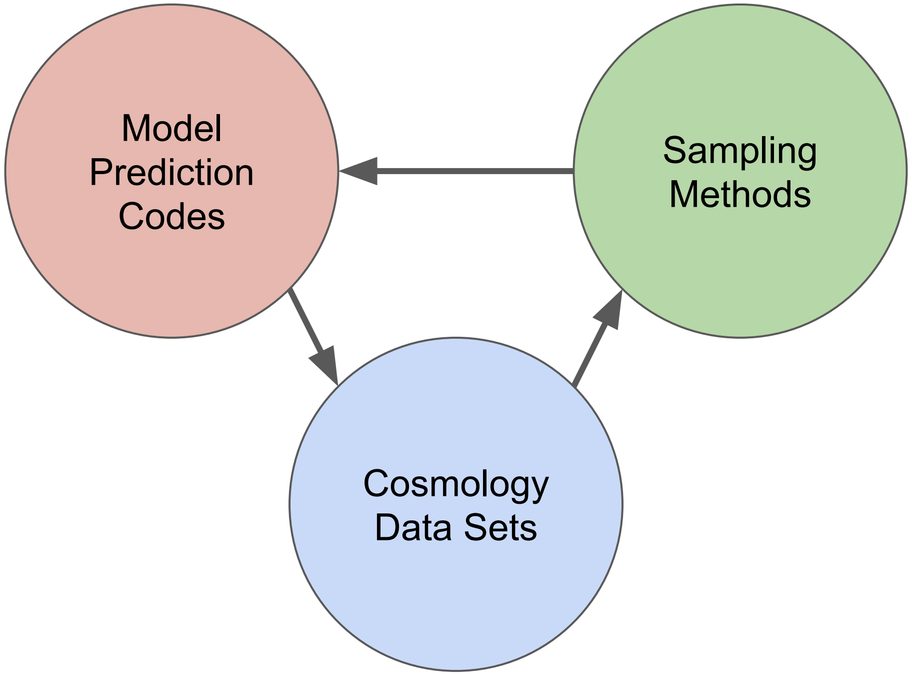
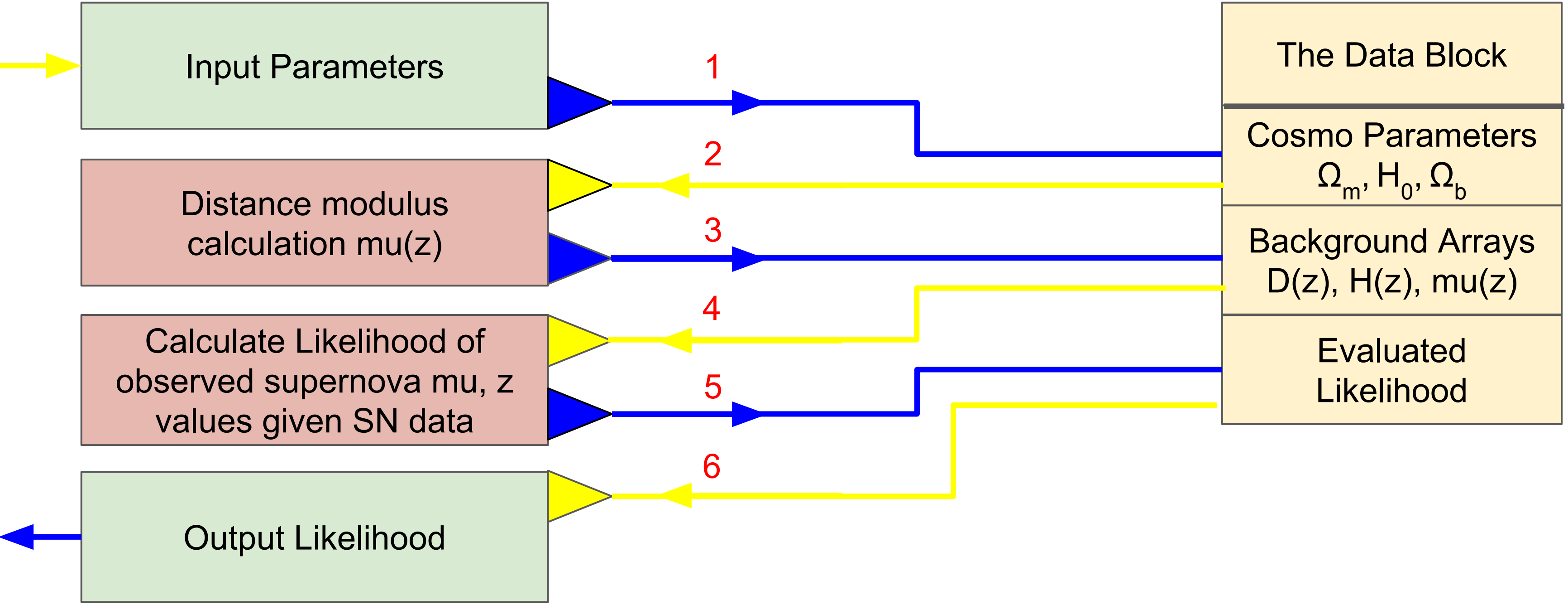
CosmoSIS goal: connecting components



Example: CosmoSIS cosmological model

Type 1A Supernova Likelihood



CosmoSIS homework

In CosmoSIS we can evaluate a single likelihood using the *test* sampler.

We will show this using CosmoSIS Demo 2 now, which generates a single CMB likelihood.

See the wiki for how to run this yourself.

Sampling methods

- Metropolis-Hastings
- Nested Sampling (Multinest)
- Ensemble Samplers (Emcee, Kombine)
- Approximate Bayesian Computation
- Grid Explorers (Snake)
- Fisher Forecasting
- Maximum Likelihoods (Minuit)

There are many sampling methods available, not specific to cosmology, with many approaches and implementations

Cosmology model prediction codes

CAMB

CLASS

CosmoLike

SNANA

MGCAMB

MGCLASS

IsItGR

EFTCamb

Colossus

CosmoCalc

AstroPy

CosmoMC

Cosmolopy

Many codes to predict different ingredients and steps in cosmological models.

Cosmology data sets

Cepheid Variables

Type IA Supernovae

Baryon Acoustic Oscillations

Strong Lensing

Light element abundances

Globular cluster ages

Cosmic Microwave Background

Redshift Space Distortions

Weak Lensing

Large-Scale Structure

Cluster Counts

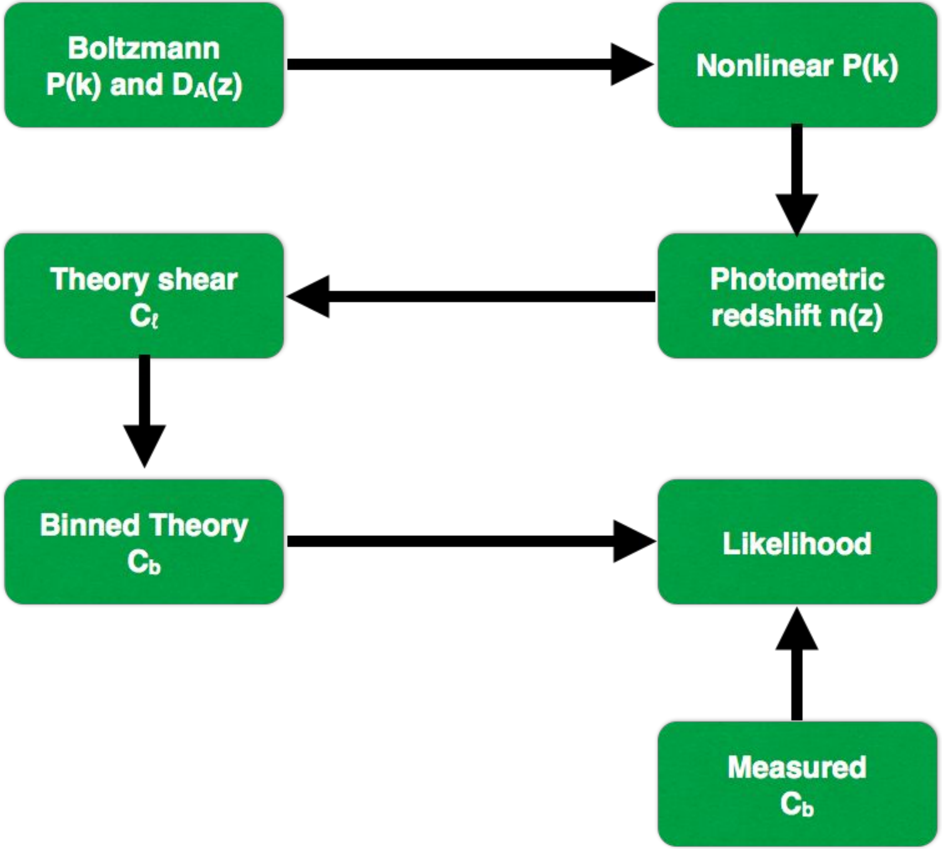
21cm line structure

Lyman Alpha Forest

Each probe is sensitive to different cosmological parameters and have different nuisance parameters

CosmoSIS: more complex example - lensing spectra

- Sequence of calculations is needed for structure likelihoods like lensing
- Like many observables, lensing depends on *matter power spectrum*
- Each module solves one complex task (if you own a state-of-the-art code - share them. Yaml files guides citation)
- General picture remains the same:
  - Solve equations to compute underlying theory prediction
  - Use stats to compute likelihood



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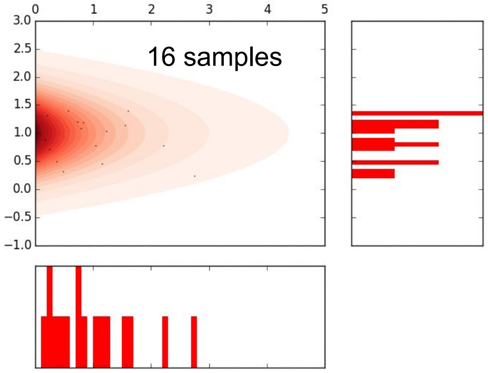
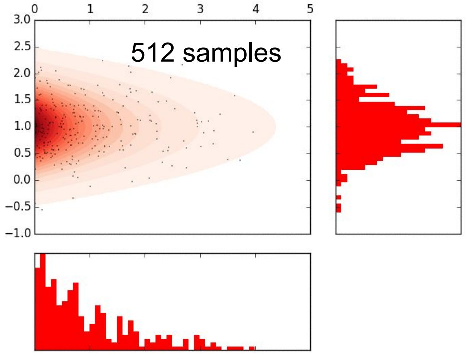
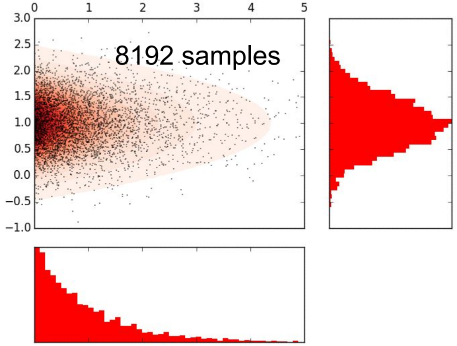
Sampling methods

We want to have a set of points the parameter space are *representative of the posterior distribution*



- Higher density <=> higher probability
- Density ratio <=> Probability ratio
- Sample moments ~ posterior moments (modulo shot noise)
- Volume where 68% of the points are located -> 68% confidence region





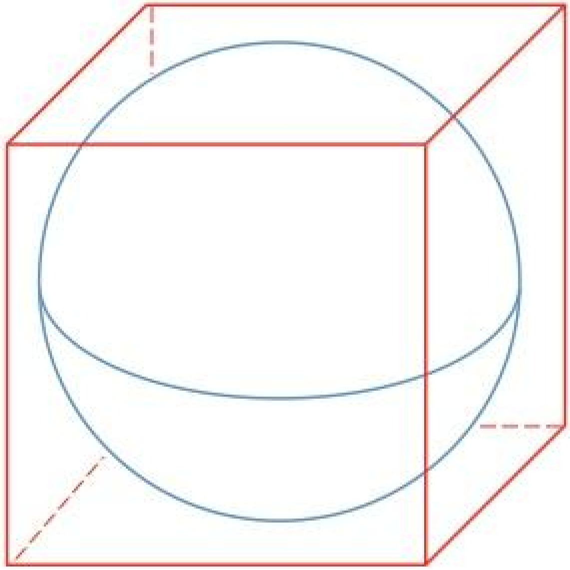
Sampling methods

We want to have a set of points the parameter space are *representative of the posterior distribution*

1. Need to efficiently explore areas of good fit

<u>This is not trivial in an high-dimensional space</u>

<u>Classical example: n-dimensional sphere contained in an n-dimensional cube</u>



$$\frac{V_n(\text{Sphere})}{V_n(\text{Cube})} \to 0, \text{ as } n \to \infty$$

We want the number of points needed to represent the posterior to not be an exponential function of the number of dimensions

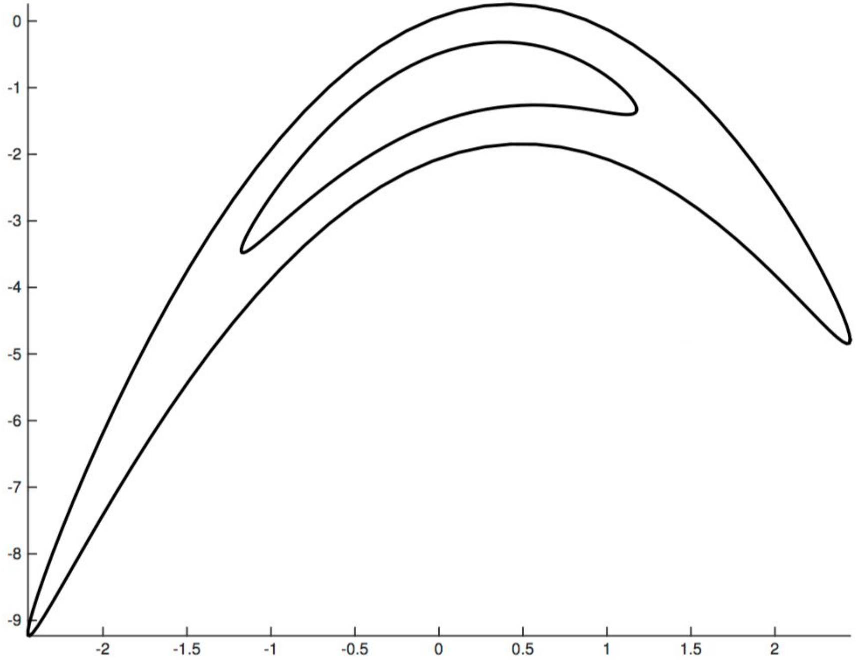
Sampling methods

We want to have a set of points the parameter space are *representative of the posterior distribution*

1. Need to efficiently explore areas of good fit

<u>This is not trivial for arbitrary shapes</u>

<u>Classical example: banana shape posterior</u>



We want methods that are robust to a variety of posterior shapes

Sampling methods

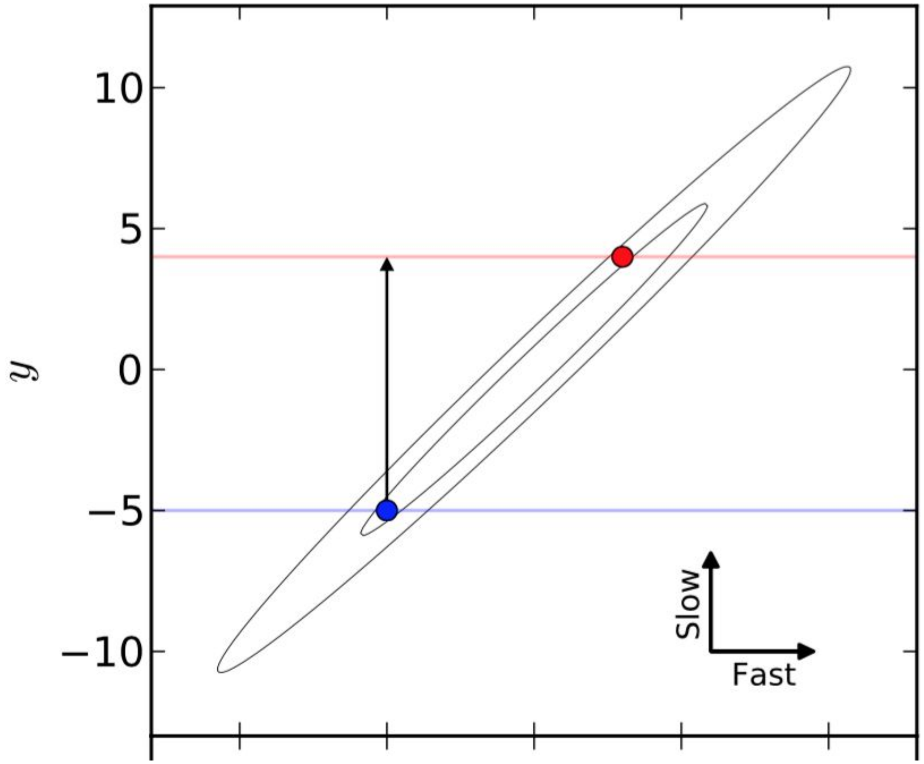
We want to have a set of points the parameter space are *representative of the posterior distribution*

1. Need to efficiently explore areas of good fit

<u>This is not trivial for arbitrary problems</u>

<u>Classical example: problems with fast/slow dimensions</u>

We will see that there are a variety of methods and no method can claim to be the best one in all situations



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- **Metropolis Hastings theory & example in CosmoSIS**
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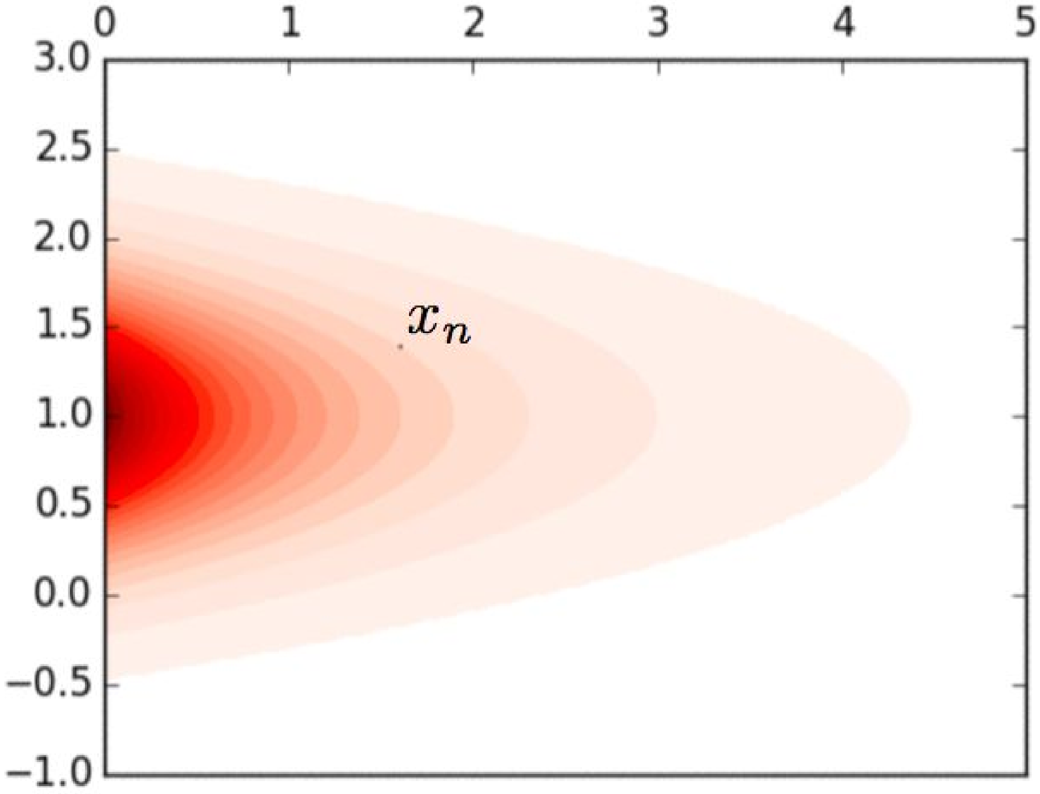
Metropolis-Hastings

Most approaches are in a class called Monte Carlo Markov Chains (MCMC)

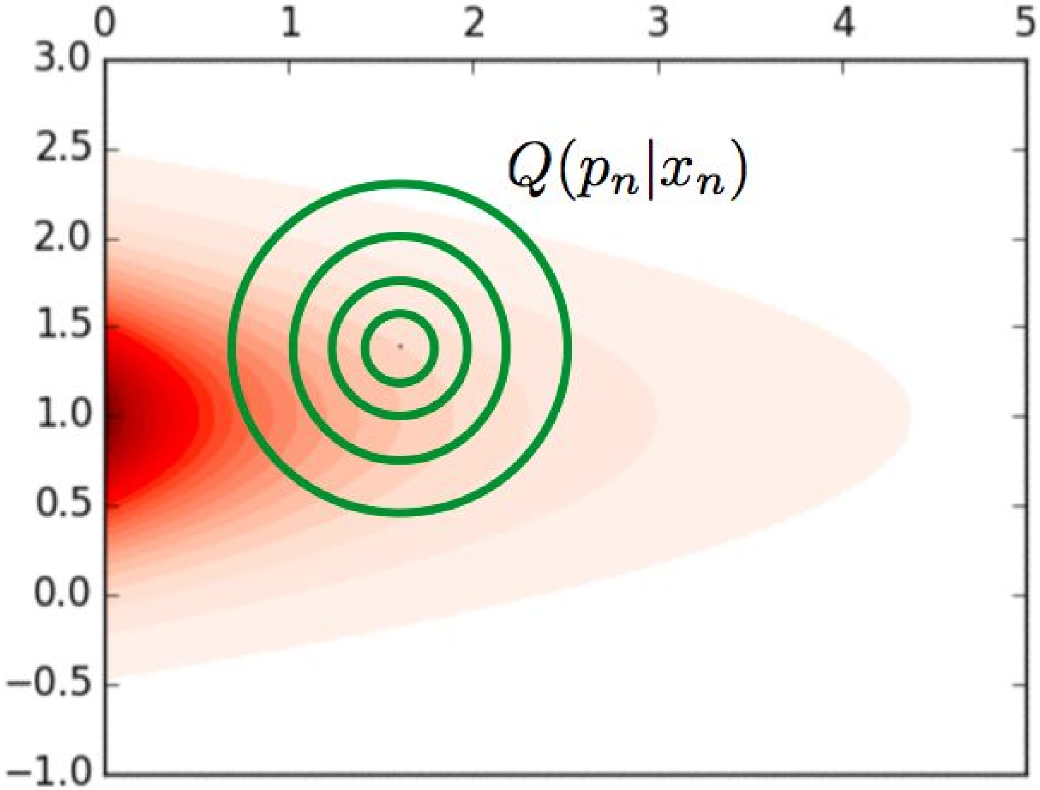
Many algorithms but important classic is *Metropolis-Hastings*



Metropolis-Hastings

We want to represent the entire posterior distribution

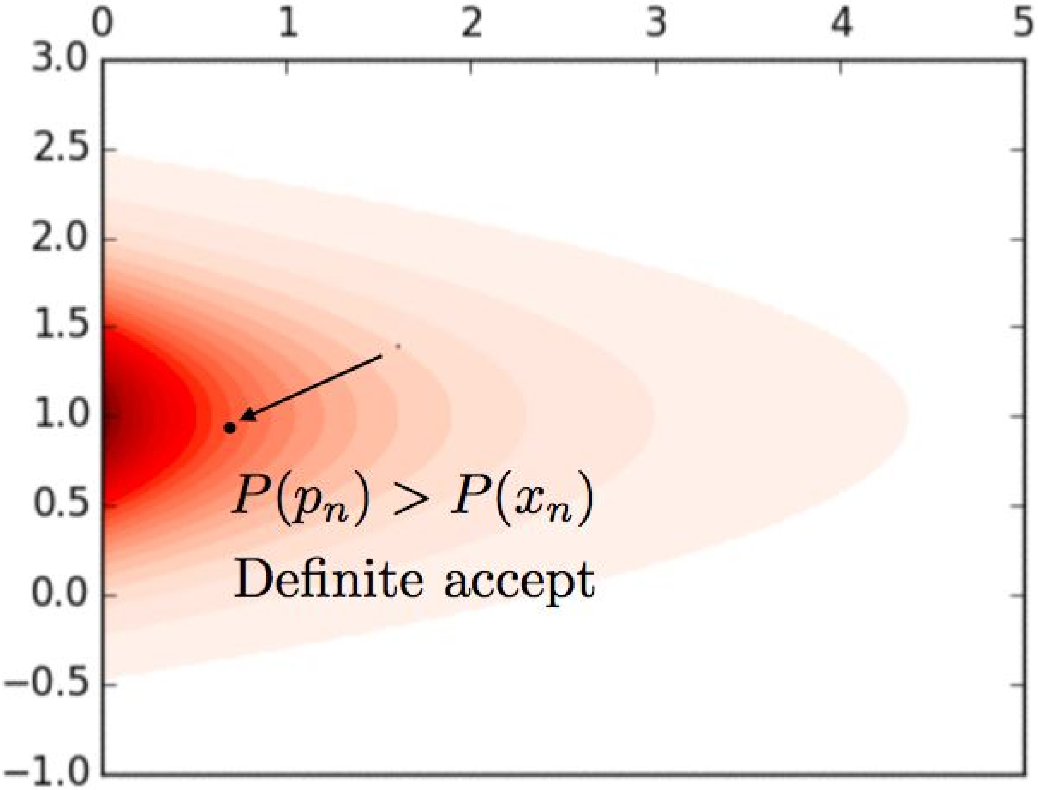
Therefore samples must be able to go from high to low probability regions



Metropolis-Hastings

We want to represent the entire posterior distribution

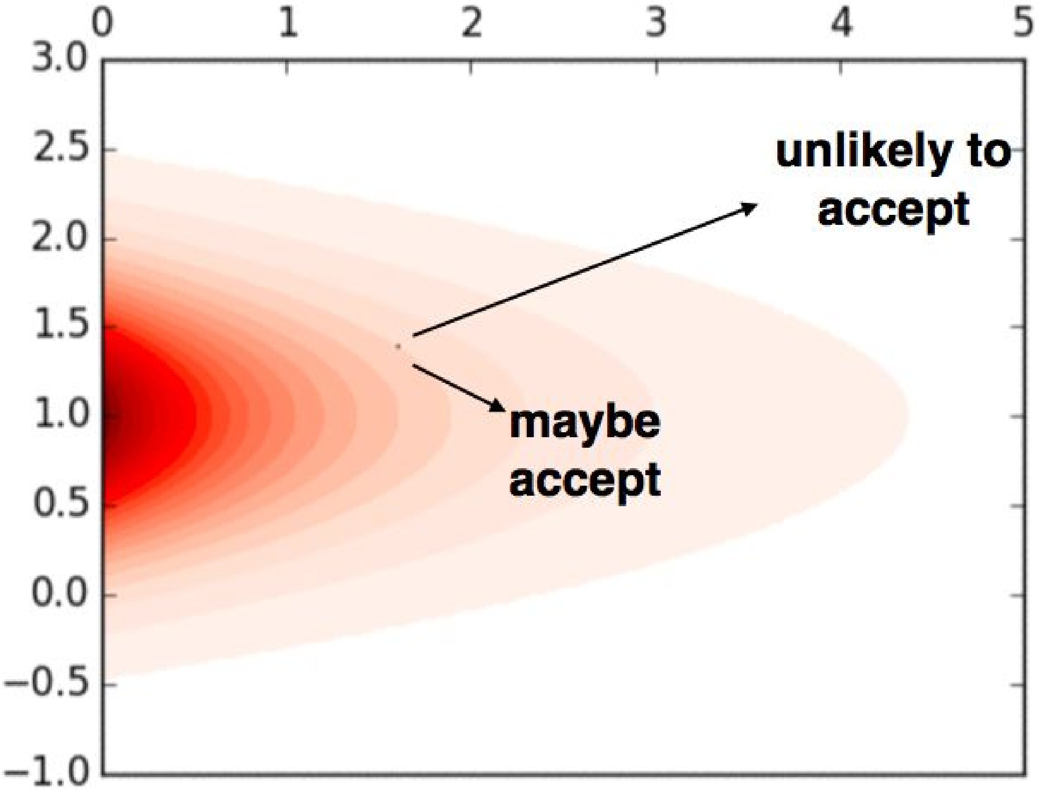
Go from low to high probability regions is always allowed.



Metropolis-Hastings

We want to represent the entire posterior distribution

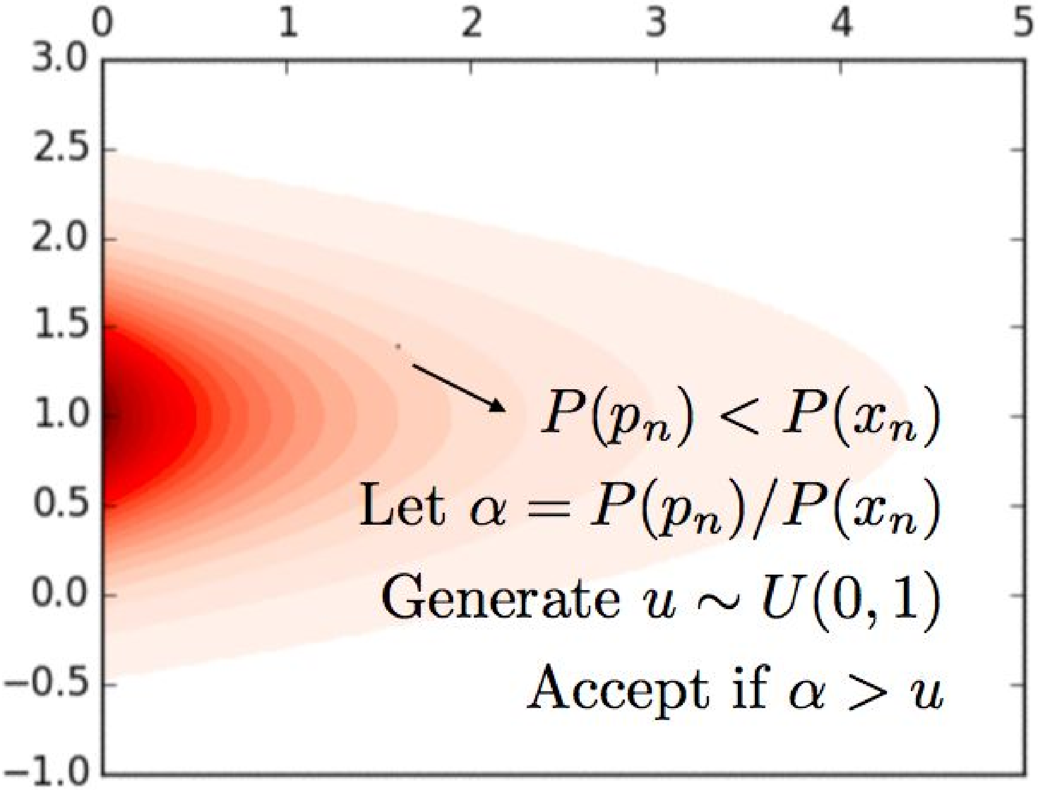
Sometimes going from high to low probability regions is allowed.



Metropolis-Hastings

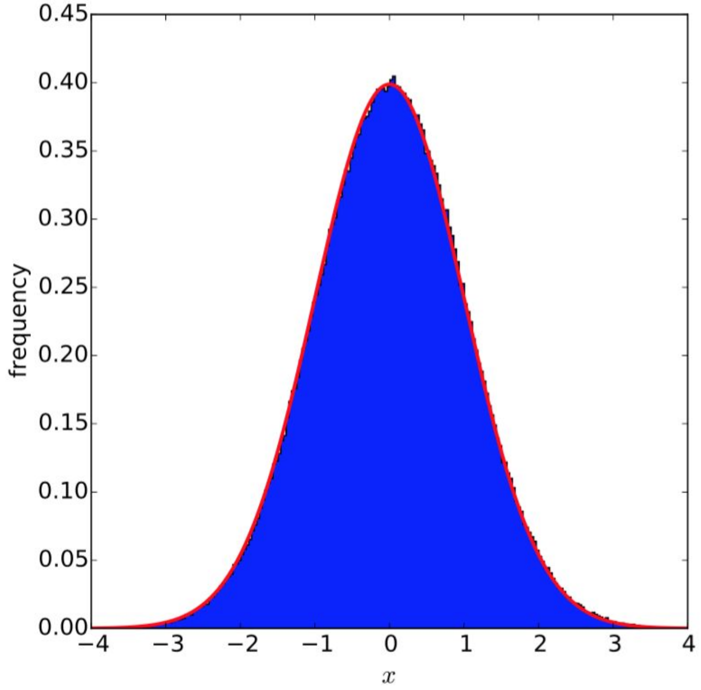
We want to represent the entire posterior distribution

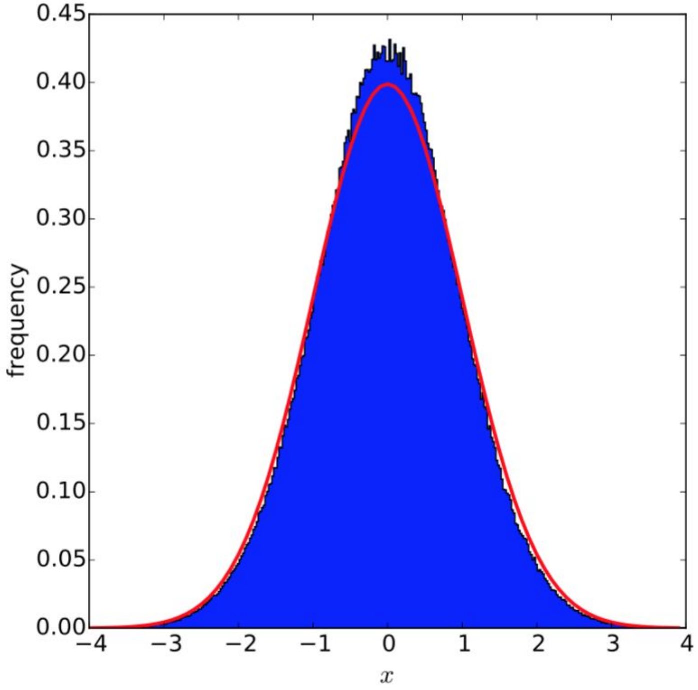
Transition from high to low probability regions depends on the exact probability ratio



Metropolis-Hastings

When a new proposed step is rejected, multiplicity of the current point must increase

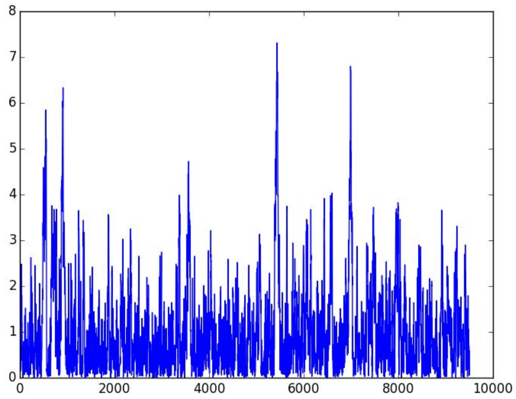


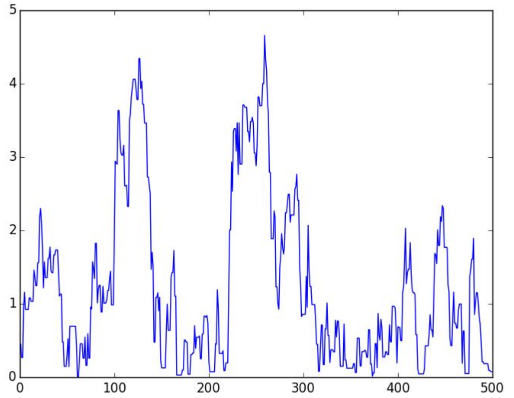


How do I know when to stop sampling?

Determining convergence is not trivial. What converge means may depend somewhat on what you want

- Look at trace plots ->
- Check Gelman-Rubin test
- Evaluate Z scores
- What if posterior is multi-modal?
- How to check errors on the contours?
- What if likelihoods based on posterior density is needed (KDE for example)?





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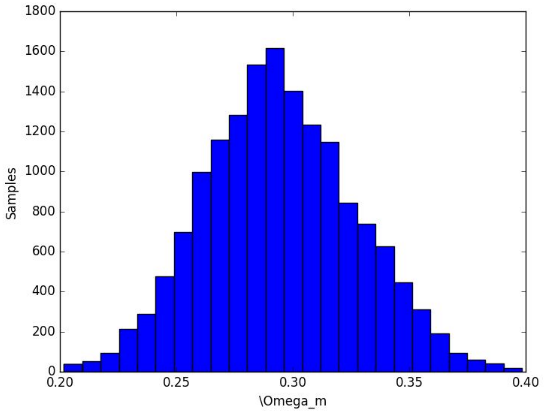
Metropolis-Hastings: CosmoSIS example

We will now switch to a terminal and run a Metropolis-Hastings example:

```
> cosmosis demos/metropolis.ini
```

Plots & Constraints from MCMC

- By construction we can make probability density plots from MCMCs by making a simple histogram in 1D or 2D
- This automatically marginalizes over the other parameters
- Can also smooth these to remove some of the scatter



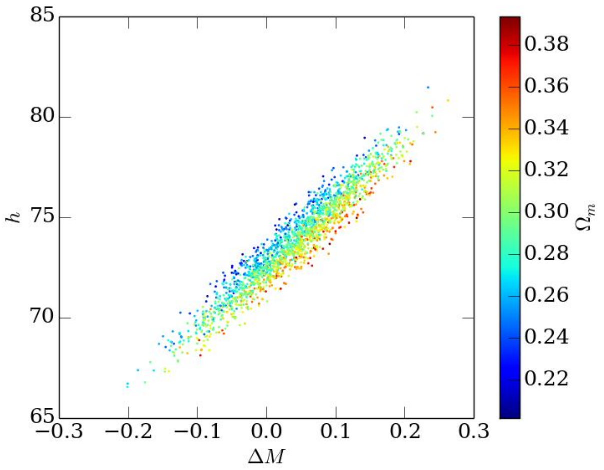
CosmoSIS Plotting

> postprocess param_file_name.ini

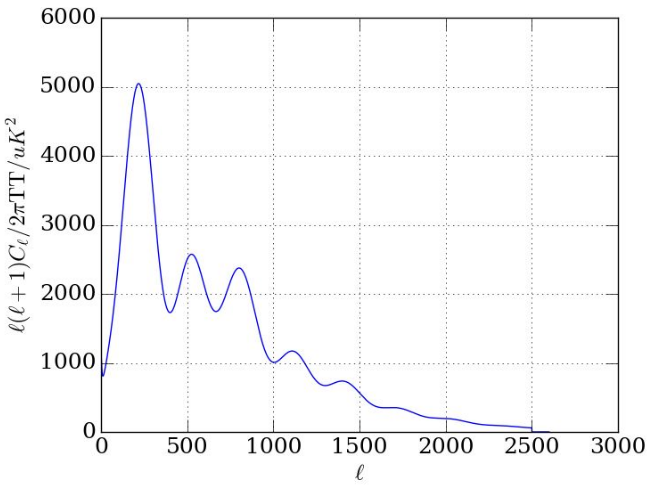
or

> postprocess chain.txt

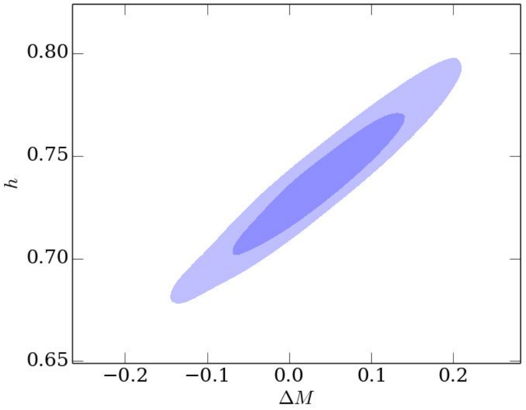
Demo 9: Scatter Plots



Demo 1: Basic Cosmology Plots



Demo 5: 2D Plots (smoothed)



More Questions & Further Reading

Demos can be found on the CosmoSIS Wiki:

https://bitbucket.org/joezuntz/cosmosis/wiki/Home